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Supersymmetric Born-Infeld from the pure spinor formalism of the open superstring

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Abstract: Classical BRST invariance in the pure spinor formalism for the open superstring is shown to imply the supersymmetric Born-Infeld equations of motion for the background fields. These equations are obtained by requiring that the left and right-moving BRST currents are equal on the worldsheet boundary in the presence of the background. The Born-Infeld equations are expressed in $N = 1$ $D = 10$ superspace and include all abelian contributions to the low-energy equations of motion, as well as the leading non-abelian contributions.

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1. Introduction

In 1934, Born and Infeld found a generalization of Maxwell theory which shares the property of being invariant under duality rotations of the electric and magnetic fields [1]. This abelian Born-Infeld theory has been supersymmetrized in $D = 4$ [2], and more recently in $D = 10$ [3, 4]. Abelian supersymmetric $D = 10$ Born-Infeld theory is uniquely determined by its invariance under $N = 2$ $D = 10$ supersymmetry, and can be deduced from the effective action of a supersymmetric D_9-brane [5]. Non-abelian supersymmetric $D = 10$ Born-Infeld theory has been discussed in various papers [6], however, there does not yet exist any complete definition of the theory.

Over fifteen years ago, it was shown that one-loop conformal invariance of the bosonic open string in an electromagnetic background implies that the background satisfies the Born-Infeld equations, and higher-loop conformal invariance implies higher-derivative corrections to these equations [7]. However, because of problems with describing fermionic backgrounds, this result was generalized only to the bosonic sector of supersymmetric Born-Infeld theory using the Ramond-Neveu-Schwarz formalism of the open superstring [8]. Although fermionic backgrounds can be classically described using the Green-Schwarz formalism of the superstring, quantization problems have prevented computation of the equations implied by one-loop or higher-loop conformal invariance. Nevertheless, it has been shown that classical κ-symmetry of the Green-Schwarz superstring in an abelian background implies the abelian supersymmetric Born-Infeld equations for the background [9, 4].

Recently, a new formalism for the superstring has been developed which is manifestly super-Poincaré covariant and does not suffer from quantization problems [10]. In this formalism, physical states are defined using the left and right-moving BRST charges

$$Q = \int d\sigma (\lambda^\alpha d_\alpha) \qquad \text{and} \qquad \widehat{Q} = \int d\sigma \left(\widehat{\lambda}^\alpha \widehat{d}_\alpha\right) \tag{1.1}$$

where d_α and $\widehat{d}_\alpha$ are left and right-moving worldsheet variables for the $N = 2$ $D = 10$ supersymmetric derivatives and λ^α and $\widehat{\lambda}^\alpha$ are left and right-moving pure spinor variables satisfying

$$\lambda\gamma^m_{\alpha\beta}\lambda^\beta = \widehat{\lambda}^\alpha\gamma^m_{\alpha\beta}\widehat{\lambda}^\beta = 0 \tag{1.2}$$

for $m = 0$ to 9. The cohomology of Q and $\widehat{Q}$ has been shown to reproduce the correct superstring spectrum [11] and scattering amplitudes computed using this formalism have been shown to agree with Ramond-Neveu-Schwarz computations [12].

As was shown in [13], classical BRST invariance of the closed superstring in a curved background implies that the background fields satisfy the full non-linear type-II supergravity equations of motion. This was verified by computing the worldsheet equations of motion for the closed superstring worldsheet variables in the presence of the curved background and showing that the BRST currents satisfy

$$\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma}\right)(\lambda^\alpha d_\alpha) = \left(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}\right)\left(\widehat{\lambda}^\alpha\widehat{d}_\alpha\right) = 0 \tag{1.3}$$

if and only if the background superfields satisfy the appropriate superspace torsion constraints and equations of motion. Since (1.3) implies that $\frac{\partial}{\partial\tau}Q = \frac{\partial}{\partial\tau}\widehat{Q} = 0$, it implies that classical BRST invariance is preserved in the presence of the closed superstring background.

In this paper, it will be shown that classical BRST invariance of the open superstring in a background implies that the background fields satisfy the full non-linear supersymmetric Born-Infeld equations of motion. This will be verified by computing the boundary conditions of the open superstring worldsheet variables in the presence of the background and showing that the left and right-moving BRST currents satisfy

$$\lambda^\alpha d_\alpha = \widehat{\lambda}^\alpha\widehat{d}_\alpha \tag{1.4}$$

on the boundary if and only if the background fields satisfy the supersymmetric Born-Infeld equations of motion. Since $\lambda^\alpha d_\alpha$ is left-moving and $\widehat{\lambda}^\alpha\widehat{d}_\alpha$ is right-moving, $\frac{\partial}{\partial\tau}(Q + \widehat{Q}) = \int d\sigma\, \frac{\partial}{\partial\sigma}(\lambda^\alpha d_\alpha - \widehat{\lambda}^\alpha\widehat{d}_\alpha)$. So (1.4) implies that classical BRST invariance is preserved in the presence of the open superstring background. So just as classical BRST invariance of the closed superstring implies the type-II supergravity equations for the background fields, classical BRST invariance of the open superstring implies the supersymmetric Born-Infeld equations for the background fields. Although similar results can be obtained using classical κ-symmetry in the Green-Schwarz formalism, this pure spinor approach has the advantage of allowing the computation of higher-derivative corrections through the requirement of quantum BRST invariance.

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To obtain at lowest order in α' the complete abelian contribution to the Born-Infeld equations, one should define the abelian component of the vector gauge field to carry dimension -1 so that the abelian vector field strength carries dimension zero. But since the non-abelian gauge field appears in the covariant derivative, gauge invariance implies that all non-abelian components of the vector gauge field must be defined to carry dimension $+1$ so that the non-abelian field strength carries dimension $+2$. With this different definition of dimension for the abelian and non-abelian gauge fields, one can consistently compute all abelian and non-abelian contributions of lowest dimension to the effective equations of motion. At lowest order in α', one obtains the complete abelian supersymmetric Born-Infeld equations, as well as Born-Infeld-like corrections to the non-abelian super-Yang-Mills equations. These Born-Infeld-like corrections to the non-abelian equations come from superstring couplings of the abelian and non-abelian gauge field and include all corrections to the non-abelian super-Yang-Mills equations which are generated by a constant abelian field strength. It should be possible to compute higher-order α' corrections to these low-energy equations of motion by performing sigma model loop computations.

Since the formalism is manifestly super-Poincaré covariant, these supersymmetric Born-Infeld equations are expressed in $N = 1$ $D = 10$ superspace. Although the lowest order contributions to the supersymmetric $D = 10$ Born-Infeld equations in superspace have been known for some time [14, 15], the complete abelian contribution to these $D = 10$ superspace equations were derived just two weeks ago in [4]. Our superspace equations were computed independently of these new results, which agree with the abelian contribution to our Born-Infeld equations. In addition to the manifest $N = 1$ $D = 10$ supersymmetry, our Born-Infeld equations are also invariant under a second supersymmetry coming from the $N = 2$ $D = 10$ supersymmetry of the closed superstring worldsheet action. This second supersymmetry contains both an abelian and non-abelian contribution.

In section 2 of this paper, the pure spinor version of the superparticle action will be reviewed and it will be shown that classical BRST invariance of the superparticle action implies super-Yang-Mills equations for the background fields. In section 3, the superparticle action will be generalized to the superstring and the boundary conditions for the open superstring worldsheet variables will be computed in the presence of an abelian background. The condition that $\lambda^\alpha d_\alpha = \widehat{\lambda}^\alpha \widehat{d}_\alpha$ on the boundary will then be shown to imply the abelian supersymmetric Born-Infeld equations in $N = 1$ $D = 10$ superspace for the abelian background superfields. In section 4, the results of section 3 will be generalized to a non-abelian background. And in section 5, our results will be summarized and the computation of higher-derivative corrections will be discussed.

2. Review of superparticle in super-Yang-Mills background

In this section, the pure spinor description of the superparticle will be reviewed and it will be shown that classical BRST invariance of the superparticle action implies the usual super-Yang-Mills equations of motion for the background superfields. These results will be generalized in later sections where it will be shown that classical BRST invariance of the

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open superstring action implies the supersymmetric Born-Infeld equations of motion for the background superfields.

2.1 Pure spinor description of the superparticle

As shown in [16], the $D=10$ superparticle can be covariantly quantized using the quadratic worldline action

$$S=\int d\tau\left(\frac{1}{2}\dot{x}^m\dot{x}_m+p_\alpha\dot{\theta}^\alpha+w_\alpha\dot{\lambda}^\alpha\right) \tag{2.1}$$

and the BRST charge

$$Q=\lambda^\alpha d_\alpha \tag{2.2}$$

where $\Pi^m=\dot{x}^m+\frac{1}{2}\theta^\alpha\gamma^m_{\alpha\beta}\dot{\theta}^\beta$ is the bosonic supersymmetric momentum, $d_\alpha=\partial_\alpha-\frac{1}{2}\Pi_m(\gamma^m\theta)_\alpha$ is the fermionic supersymmetric momentum, and λ^α is a bosonic spinor satisfying the pure spinor constraint $\lambda\gamma^m\lambda=0$ for $m=0$ to 9. Because of the pure spinor constraint on λ^α, its conjugate momentum w_α is only defined up to the gauge transformation $w_\alpha\sim w_\alpha+\Lambda_m(\gamma^m\lambda)_\alpha$ for arbitrary gauge parameter Λ_m. Note that one can non-covariantly express λ^α in terms of independent variables, however, this will not be necessary in this paper.

Physical states in this formalism are described by vertex operators of ghost-number one in the cohomology of Q. Since only λ^α carries ghost-number, the vertex operator at ghost-number one is $V=\lambda^\alpha A_\alpha(x,\theta)$ where $A_\alpha(x,\theta)$ is an $N=1$ $D=10$ superfield. And since $QV=\lambda^\alpha\lambda^\beta D_\alpha A_\beta=\frac{1}{3840}(\lambda\gamma^{mnpqr}\lambda)(D\gamma_{mnpqr}A)$ where $D_\alpha=\frac{\partial}{\partial\theta^\alpha}+\frac{1}{2}(\gamma^m\theta)_\alpha\partial_m$ is the $N=1$ $D=10$ supersymmetric derivative, $QV=0$ implies that $D\gamma^{mnpqr}A=0$ for any five-form direction $mnpqr$. Also, $\delta V=Q\Lambda(x,\theta)=\lambda^\alpha D_\alpha\Lambda$ implies that $\delta A_\alpha=D_\alpha\Lambda$. As will now be reviewed, these are the linearized super-Yang-Mills equations of motion and gauge invariances expressed in terms of the spinor superfield A_α.

To show that A_α describes linearized super-Yang-Mills, use $\Lambda(x,\theta)=h_\alpha(x)\theta^\alpha+j_{\alpha\beta}(x)\theta^\alpha\theta^\beta$ to gauge away $A_\alpha|_{\theta=0}$ and the three-form part of $(D_\alpha A_\beta)|_{\theta=0}$. Since $D\gamma^{mnpqr}\times A=0$ implies that the five-form part of $(D_\alpha A_\beta)|_{\theta=0}$ vanishes, the lowest non-vanishing component of $A_\alpha(x,\theta)$ in this gauge is the vector component $(D\gamma_m A)|_{\theta=0}$. Continuing this type of argument to higher order in θ^α, one finds that there exists a gauge choice such that

$$A_\alpha(x,\theta)=(\gamma^m\theta)_\alpha a_m(x)+(\theta\gamma^{mnp}\theta)(\gamma_{mnp})_{\alpha\beta}\chi^\beta(x)+\dots \tag{2.3}$$

where $a_m(x)$ and $\chi^\beta(x)$ satisfy the linearized super-Yang-Mills equations of motion $\partial^m\partial_{[m}a_{n]}=\gamma^m_{\alpha\beta}\partial_m\chi^\beta=0$ and the component fields in $\dots$ are spacetime derivatives of $a_m(x)$ and $\chi^\beta(x)$.

2.2 Superparticle in super-Yang-Mills background

Just as the relativistic particle action can be generalized in a Yang-Mills background, the superparticle action of (2.1) can be generalized in a super-Yang-Mills background. This action is defined as

$$S=\int d\tau\left(\frac{1}{2}\dot{x}^m\dot{x}_m+p_\alpha\dot{\theta}^\alpha+w_\alpha\dot{\lambda}^\alpha+\bar{\eta}^I\nabla\eta_I\right) \tag{2.4}$$

where $[\eta_I, \bar\eta^J]$ are complex worldline fermions whose indices I and J go from 1 to N,

$$\nabla\eta_I = \dot\eta_I + \left[\dot\theta^\alpha A_{\alpha I}{}^J(x,\theta) + \Pi^m B_{mI}{}^J(x,\theta) + d_\alpha W_I{}^{\alpha J}(x,\theta) + \frac{1}{2}N^{mn}F_{mnI}{}^J(x,\theta)\right]\eta_J,$$

$[A_\alpha, B_m, W^\alpha, F_{mn}]$ are background super-Yang-Mills superfields with gauge group U(N), and $N_{mn} = \frac{1}{2}\lambda\gamma_{mn}w$ is the Lorentz current for the pure spinor variables. For SO(N) gauge group, the complex worldline fermions should be replaced by real fermions η^I for $I = 1$ to N. And for an abelian gauge group, the worldline fermions can be omitted from the action.

As in [17], the background couplings in (2.4) can be understood geometrically as covariantization of the superparticle worldline variables where

$$\dot\theta^\alpha \to \dot\theta^\alpha + W^\alpha\,, \quad \Pi_m \to \Pi_m + B_m\,, \quad d_\alpha \to d_\alpha - A_\alpha\,, \quad N_{mn} \to N_{mn} + F_{mn}\,. \tag{2.5}$$

Note that the action of (2.4) is invariant under the gauge transformation

$$\begin{aligned} \delta A_\alpha &= D_\alpha\Lambda + [A_\alpha, \Lambda]\,, \quad \delta B_m = \partial_m\Lambda + [B_m, \Lambda]\,, \quad \delta W^\alpha = [W^\alpha, \Lambda]\,, \quad \delta F^{mn} = [F^{mn}, \Lambda]\,, \\ \delta\eta_I &= -\Lambda_I{}^J\eta_J\,, \quad \delta\bar\eta^I = \bar\eta^J\Lambda_J{}^I\,. \end{aligned} \tag{2.6}$$

As will now be shown, the superparticle action of (2.4) is classically BRST invariant when the background superfields $[A_\alpha, B_m, W^\alpha, F^{mn}]$ satisfy the super-Yang-Mills equations where A_α and B_m are the spinor and vector gauge superfields and W^α and F^{mn} are the spinor and vector superfield strengths.

The simplest way to find the conditions implied by classical BRST invariance of (2.4) is to require that the BRST charge is conserved, i.e. that $\dot Q = \frac{\partial}{\partial\tau}(\lambda^\alpha d_\alpha) = 0$. By varying $w_\alpha \to w_\alpha + \delta w_\alpha$ in the action, one finds the equation of motion

$$\dot\lambda^\alpha = \frac{1}{4}\bar\eta^I\eta_J(\gamma^{mn}\lambda)^\alpha F_{mnI}{}^J\,. \tag{2.7}$$

And by varying $\theta^\alpha \to \theta^\alpha + \delta\theta^\alpha$ and $x^m \to x^m - \frac{1}{2}\theta\gamma^m\delta\theta$ in the action, one finds the equation of motion

$$\begin{aligned} \dot d_\alpha &= \frac{\partial}{\partial\tau}\left(\bar\eta^I\eta_J A_{\alpha I}{}^J\right) + \bar\eta^I\eta_J\Big[-\dot\theta^\beta D_\alpha A_\beta + \Pi^m D_\alpha B_m - d_\beta D_\alpha W^\beta + \\ &\qquad\qquad + \frac{1}{2}N_{mn}D_\alpha F^{mn} + (\gamma^m\dot\theta)_\alpha B_m + \Pi_m(\gamma^m W)^\alpha\Big]_I{}^J \\ &= \bar\eta^J\eta_I\Big[\dot\theta^\beta\left(D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\} - \gamma^m_{\alpha\beta}B_m\right) + \\ &\qquad + \Pi^m\left(\partial_m A_\alpha - D_\alpha B_m + [B_m, A_\alpha] + \gamma_{m\alpha\beta}W^\beta\right) + \\ &\qquad + d_\beta\left(D_\alpha W^\beta + \left[A_\alpha, W^\beta\right]\right) - \frac{1}{2}N_{mn}\left(D_\alpha F^{mn} + [A_\alpha, F^{mn}]\right)\Big]_I{}^J\,, \end{aligned} \tag{2.8}$$

where the equations of motion for $\bar\eta^I$ and η_J have been used.

So putting together (2.7) and (2.8), $\dot Q = 0$ implies that the background superfields satisfy

$$\begin{aligned} D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\} &= \gamma^m_{\alpha\beta}B_m, \\ \partial_m A_\alpha - D_\alpha B_m + [B_m, A_\alpha] &= -\gamma_{m\alpha\beta}W^\beta \\ D_\alpha W^\beta + \left[A_\alpha, W^\beta\right] &= \frac{1}{4}(\gamma_{mn})_\alpha{}^\beta F^{mn}, \\ \lambda^\alpha\lambda^\beta\,(\gamma_{mn})_\beta{}^\gamma(D_\alpha F^{mn} + [A_\alpha, F^{mn}]) &= 0\,. \end{aligned} \tag{2.9}$$

The equations of (2.9) will now be shown to describe super-Yang-Mills where $[A_\alpha, B_m]$ are the gauge superfields and $[W^\alpha, F^{mn}]$ are the field strengths.

If the first equation of (2.9) is contracted with $\gamma^{\alpha\beta}_{mnpqr}$, one obtains

$$\gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + A_\alpha A_\beta) = 0 \tag{2.10}$$

which is the non-abelian super-Yang-Mills equation expressed in terms of a spinor superfield. Contracting the first equation of (2.9) with $\gamma^{\alpha\beta}_m$ defines

$$B_m = \frac{1}{8}\gamma^{\alpha\beta}_m(D_\alpha A_\beta + A_\alpha A_\beta)\,, \tag{2.11}$$

which is the standard definition of the super-Yang-Mills vector gauge superfield.

Contracting the second equation of (2.9) with $\gamma^{m\alpha\gamma}$ implies that

$$W^\gamma = \frac{1}{10}\gamma^{m\alpha\gamma}(D_\alpha B_m - \partial_m A_\alpha + [A_\alpha, B_m])\,, \tag{2.12}$$

which is the standard definition of the spinor field strength. And the gamma-matrix traceless part of the second equation of (2.9) is implied through Bianchi identities by the first equation of (2.9). Contracting the third equation of (2.9) with $(\gamma^{pq})_\alpha{}^\beta$ implies that

$$F^{pq} = \;-\frac{1}{8}(\gamma^{pq})_\alpha{}^\beta \nabla_\beta W^\alpha \tag{2.13}$$

where $\nabla_\alpha = D_\alpha + A_\alpha$ is the covariant spinor derivative, and other contractions of the third equation are implied through Bianchi identities from the first two equations. Using (2.10) -(2.12), (2.13) implies that F_{mn} can also be written as $F_{mn} = \partial_m B_n - \partial_n B_m + [B_m, B_n]$, which is the standard definition of the vector field strength. Finally, the last equation of (2.9) is implied by the first three equations since λ^α being a pure spinor implies that $\lambda^\alpha\lambda^\beta\nabla_\alpha\nabla_\beta W^\gamma = 0$.

So it has been shown that classical BRST invariance of the superparticle action of (2.4) implies the super-Yang-Mills equations of motion for the background superfields. In the next sections, this result will be generalized to the open superstring where classical BRST invariance will imply the supersymmetric Born-Infeld equations for the background superfields.

3. Open superstring in abelian background

In this section, it will be shown that classical BRST invariance of the open superstring in an abelian background implies the abelian supersymmetric Born-Infeld equations of motion for the background superfields. The first step in computing the equations implied by classical BRST invariance is to determine the appropriate boundary conditions for the open superstring worldsheet variables in the presence of the background. Recall that for the bosonic string in an electromagnetic background, the Neumann boundary conditions $\frac{\partial}{\partial\sigma}x^m = 0$ are modified to

$$\frac{\partial}{\partial\sigma}x^m = F^{mn}\dot{x}^n \tag{3.1}$$

where F^{mn} is the electromagnetic field strength. For the bosonic string, these modified boundary conditions do not affect classical BRST invariance since (3.1) together with $F^{mn} = -F^{nm}$ implies that the left-moving stress-tensor $T = \frac{1}{2}\partial x^m \partial x_m$ remains equal to the right-moving stress-tensor $\widehat{T} = \frac{1}{2}\bar{\partial} x^m \bar{\partial} x_m$ on the boundary where $\partial = \frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}$ and $\bar{\partial} = \frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma}$. So by defining the left and right-moving reparameterization ghosts to satisfy $c = \widehat{c}$ and $b = \widehat{b}$ on the boundary, one is guaranteed that the left and right-moving BRST currents coincide on the boundary in the presence of the background.

However, for the superstring using the pure spinor formalism, the boundary conditions on the worldsheet variables in the presence of a background do not automatically imply that the left and right-moving BRST currents coincide on the boundary. As will be shown in the following subsections, $\lambda^\alpha d_\alpha = \widehat{\lambda}^\alpha \widehat{d}_\alpha$ on the boundary if and only if the background superfields satisfy the supersymmetric Born-Infeld equations of motion.

3.1 Review of free open superstring using pure spinor formalism

The quadratic superparticle action of (2.1) is easily generalized to the superstring action in conformal gauge

$$S_0 = -\frac{1}{\alpha'}\int d\tau d\sigma \left\{ \frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + \widehat{p}_\alpha \partial \widehat{\theta}^\alpha + w_\alpha \bar{\partial}\lambda^\alpha + \widehat{w}_\alpha \partial \widehat{\lambda}^\alpha \right\} \tag{3.2}$$

where $(\theta^\alpha, p_\alpha, \lambda^\alpha, w_\alpha)$ are left-moving variables, $(\widehat{\theta}^\alpha, \widehat{p}_\alpha, \widehat{\lambda}^\alpha, \widehat{w}_\alpha)$ are right-moving variables, and λ^α and $\widehat{\lambda}^\alpha$ are pure spinor variables satisfying $\lambda\gamma^m\lambda = \widehat{\lambda}\gamma^m\widehat{\lambda} = 0$.

For the closed superstring, all worldsheet variables are periodic and the action of (3.2) is invariant under the $N = 2$ $D = 10$ spacetime supersymmetry transformations

$$\begin{aligned} \delta\theta^\alpha &= \epsilon^\alpha , \quad \delta\widehat{\theta}^\alpha = \widehat{\epsilon}^\alpha , \quad \delta x^m = \frac{1}{2}\theta\gamma^m\epsilon + \frac{1}{2}\widehat{\theta}\gamma^m\widehat{\epsilon} , \\ \delta p_\alpha &= \frac{1}{2}\gamma^m_{\alpha\beta}\partial x_m \epsilon^\beta - \frac{1}{8}\gamma^m_{\alpha\delta}\gamma_{m\beta\gamma}\epsilon^\beta\theta^\gamma\partial\theta^\delta , \quad \delta\widehat{p}_\alpha = \frac{1}{2}\gamma^m_{\alpha\beta}\bar{\partial} x_m \widehat{\epsilon}^\beta - \frac{1}{8}\gamma^m_{\alpha\delta}\gamma_{m\beta\gamma}\widehat{\epsilon}^\beta\widehat{\theta}^\gamma\bar{\partial}\widehat{\theta}^\delta . \end{aligned} \tag{3.3}$$

Note that $[\lambda^\alpha, w_\alpha, \widehat{\lambda}^\alpha, \widehat{w}_\alpha]$ are invariant under (3.3) and the cubic terms in the transformation of p_α and $\widehat{p}_\alpha$ are needed so that $[\delta_{\epsilon_1}, \delta_{\epsilon_2}]p_\alpha = 0$ and $[\delta_{\widehat{\epsilon}_1}, \delta_{\widehat{\epsilon}_2}]\widehat{p}_\alpha = 0$. Left and right-moving supersymmetric invariants on the worldsheet can be defined as

$$\begin{aligned} &\partial\theta^\alpha , \quad \Pi^m = \partial x^m + \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\alpha\partial\theta^\beta , \quad d_\alpha = p_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\beta\partial x_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\theta^\beta\theta^\gamma\partial\theta^\delta , \\ &\bar{\partial}\widehat{\theta}^\alpha , \quad \widehat{\Pi}^m = \bar{\partial} x^m + \frac{1}{2}\gamma^m_{\alpha\beta}\widehat{\theta}^\alpha\bar{\partial}\widehat{\theta}^\beta , \quad \widehat{d}_\alpha = \widehat{p}_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\widehat{\theta}^\beta\bar{\partial} x_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\widehat{\theta}^\beta\widehat{\theta}^\gamma\bar{\partial}\widehat{\theta}^\delta , \end{aligned} \tag{3.4}$$

and the left and right-moving BRST charges are defined as

$$Q = \int d\sigma\, \lambda^\alpha d_\alpha , \qquad \widehat{Q} = \int d\sigma\, \widehat{\lambda}^\alpha \widehat{d}_\alpha . \tag{3.5}$$

For the open superstring with Neumann boundary conditions $\frac{\partial}{\partial\sigma}x^m = 0$, the surface term equations of motion from varying the worldsheet variables in (3.2) imply that

$$p_\alpha \delta\theta^\alpha - \widehat{p}_\alpha \delta\widehat{\theta}^\alpha + w_\alpha \delta\lambda^\alpha - \widehat{w}_\alpha \delta\widehat{\lambda}^\alpha = 0 \tag{3.6}$$

on the boundary. If one requires in addition that $\lambda^\alpha d_\alpha = \widehat{\lambda}^\alpha \widehat{d}_\alpha$ on the boundary, the only two consistent choices for boundary conditions of the worldsheet variables are either

$$\partial x^m = \bar{\partial} x^m \,, \quad \theta^\alpha = \widehat{\theta}^\alpha \,, \quad p_\alpha = \widehat{p}_\alpha \,, \quad \lambda^\alpha = \widehat{\lambda}^\alpha \,, \quad w_\alpha = \widehat{w}_\alpha \,, \tag{3.7}$$

or

$$\partial x^m = \bar{\partial} x^m \,, \quad \theta^\alpha = -\widehat{\theta}^\alpha \,, \quad p_\alpha = -\widehat{p}_\alpha \,, \quad \lambda^\alpha = -\widehat{\lambda}^\alpha \,, \quad w_\alpha = -\widehat{w}_\alpha \,. \tag{3.8}$$

The first choice corresponds to D_9-brane boundary conditions and the second choice corresponds to D_9-antibrane boundary conditions. If one had chosen $9-p$ of the x^m variables to satisfy Dirichlet boundary conditions, the conditions of (3.7) and (3.8) would be modified to the appropriate D_p-brane or D_p-antibrane boundary conditions. In the discussion that follows, we shall only consider the D_9-brane boundary conditions of (3.7) and will compute modifications to these conditions in the presence of background fields.

3.2 Manifest $N = 1$ $D = 10$ supersymmetry

In the presence of a background, the free boundary conditions of the superstring worldsheet variables of (3.7) are modified in a manner analagous to the bosonic string boundary conditions of (3.1). To find the appropriate boundary conditions, it is convenient to first define linear combinations of the worldsheet variables as

$$\theta^\alpha_\pm = \frac{1}{\sqrt{2}}(\theta^\alpha \pm \widehat{\theta}^\alpha) \,, \quad p^\pm_\alpha = \sqrt{2}(p_\alpha \pm \widehat{p}_\alpha) \,, \quad \lambda^\alpha_\pm = \frac{1}{\sqrt{2}}(\lambda^\alpha \pm \widehat{\lambda}^\alpha) \,, \quad w^\pm_\alpha = \sqrt{2}(w_\alpha \pm \widehat{w}_\alpha) \,. \tag{3.9}$$

Note that the free boundary conditions of (3.7) are invariant under the supersymmetry transformations parameterized by (3.3) when ϵ^α is set equal to $\widehat{\epsilon}^\alpha$. Under this $N = 1$ $D = 10$ supersymmetry, the variables of (3.9) transform as

$$\delta_{\epsilon_+} \theta^\alpha_+ = \epsilon^\alpha_+ \,, \quad \delta_{\epsilon_+} \theta^\alpha_- = 0 \,, \quad \delta_{\epsilon_+} x^m = \frac{1}{2} \theta_+ \gamma^m \epsilon_+ \tag{3.10}$$

where $\epsilon^\alpha_+ = \frac{1}{\sqrt{2}}(\epsilon^\alpha + \widehat{\epsilon}^\alpha)$, and the transformation $\delta_{\epsilon_+} p^\pm_\alpha$ can be determined from (3.3).

To preserve $N = 1$ $D = 10$ supersymmetry, one would like the modified boundary conditions in the presence of the background to also be invariant under the $N = 1$ $D = 10$ transformations of (3.10). Note that under the $N = 2$ $D = 10$ supersymmetry transformation of (3.3),

$$\delta S_0 = \frac{1}{\alpha'} \int d\tau \left\{ \frac{1}{4} \left(\widehat{\epsilon} \gamma^m \widehat{\theta} - \epsilon \gamma^m \theta \right) \dot{x}_m + \frac{1}{24} \epsilon \gamma^m \theta \, \dot{\theta} \gamma_m \theta - \frac{1}{24} \widehat{\epsilon} \gamma^m \widehat{\theta} \, \dot{\widehat{\theta}} \gamma_m \widehat{\theta} \right\} . \tag{3.11}$$

Although $\delta S_0 = 0$ when $\epsilon = \widehat{\epsilon}$ using the free boundary conditions of (3.7), δS_0 does not vanish when $\epsilon = \widehat{\epsilon}$ for arbitrary boundary conditions. However, as will now be shown, the variation of S_0 under (3.10) can be cancelled by adding to the action the surface term

$$S_b = \frac{1}{2\alpha'} \int d\tau \Bigg(\frac{1}{2} \Pi^m_+ (\theta_+ \gamma_m \theta_-) + \frac{1}{8} \left(\dot{\theta}_+ \gamma \theta_+ \right) (\theta_+ \gamma \theta_-) + \frac{1}{24} \left(\dot{\theta}_- \gamma \theta_- \right) (\theta_- \gamma \theta_+) + \\ + c_1 d^+_\alpha \theta^\alpha_- + c_2 w^+_\alpha \lambda^\alpha_- \Bigg) \tag{3.12}$$

where

$$\Pi_+^m = \dot{x}^m + \frac{1}{2}\gamma^m_{\alpha\beta}\theta_+^\alpha\dot{\theta}_+^\beta\,,$$
$$d_\alpha^+ = p_\alpha^+ - \frac{1}{2}\gamma^m_{\alpha\beta}\theta_+^\alpha\dot{x}_m - \frac{1}{8}\gamma_{\alpha\beta}\gamma_{\gamma\delta}\theta_+^\beta\theta_+^\gamma\dot{\theta}_+^\delta + \frac{1}{8}\gamma_{\alpha\beta}\gamma_{\gamma\delta}\theta_+^\gamma\theta_-^\delta\dot{\theta}_-^\beta + \frac{1}{8}\gamma_{\alpha\beta}\gamma_{\gamma\delta}\theta_-^\beta\theta_-^\gamma\dot{\theta}_+^\delta\,, \quad (3.13)$$

and c_1 and c_2 are constants which will be discussed later. Since

$$\Pi_+^m = \Pi^m + \widehat{\Pi}^m - \frac{1}{2}\theta_-\gamma^m\dot{\theta}_-\,, \quad d_\alpha^+ = \sqrt{2}\left(d_\alpha + \widehat{d}_\alpha\right) + \frac{1}{2}\gamma_{m\alpha\beta}\theta_-^\beta\left(\Pi^m - \widehat{\Pi}^m\right), \quad (3.14)$$

and since θ_-^α is invariant under (3.10), Π_+^m and d_α^+ are also invariant under (3.10). Using this invariance, one can easily check that $\delta_{\epsilon_+}(S_0+S_b) = 0$ for arbitrary boundary conditions of the worldsheet variables. Although the terms $c_1 d_\alpha^+\theta_-^\alpha + c_2 w_\alpha^+\lambda_-^\alpha$ in S_b are separately invariant under (3.10), it will be seen later that c_1 and c_2 must be non-zero in order to define consistent boundary conditions in the presence of background fields.

3.3 Boundary conditions in an abelian background

As in the superparticle action, the abelian background superfields couple in the open superstring action as $S = S_0 + S_b + V$ where

$$V = \frac{1}{2\alpha'}\int d\tau\left(\dot{\theta}_+^\alpha A_\alpha(x,\theta_+) + \Pi_+^m B_m(x,\theta_+) + d_\alpha^+ W^\alpha(x,\theta_+) + \frac{1}{2}N_+^{mn}F_{mn}(x,\theta_+)\right) \quad (3.15)$$

and $N_+^{mn} = \frac{1}{2}\lambda_+\gamma^{mn}w^+$. Note that $[\theta^\alpha, x^m, p_\alpha, N^{mn}]$ carry dimension $[-\frac{1}{2}, -1, -\frac{3}{2}, -2]$, so the abelian background superfields $[A_\alpha, B_m, W^\alpha, F_{mn}]$ carry dimension $[-\frac{3}{2}, -1, -\frac{1}{2}, 0]$ as explained in the introduction. Since the superfields in V are functions of x^m and θ_+^α which transform covariantly under (3.10), the action $S = S_0 + S_b + V$ is manifestly invariant under this $N = 1$ $D = 10$ supersymmetry.

Since S_b and V are surface terms, the equations of motion in the bulk for the worldsheet variables are the same as in the quadratic action S_0. However, the surface term equations of motion coming from S_b and V will modify the surface term equations of motion of (3.6). Defining $\delta y^m = \delta x^m - \frac{1}{2}\gamma^m_{\alpha\beta}\delta\theta_+^\alpha\theta_+^\beta$ and $D_\alpha = \frac{\partial}{\partial\theta_+^\alpha} + \frac{1}{2}\gamma^m_{\alpha\beta}\theta_+^\beta\frac{\partial}{\partial x^m}$, one finds that the surface variation of the action and the vertex are

$$\begin{aligned}\delta(S_0+S_b) = \frac{1}{2\alpha'}\int d\tau\,\Big\{&\delta\theta_+^\alpha\left[\sqrt{2}(d_\alpha - \widehat{d}_\alpha) + \gamma^m_{\alpha\beta}\theta_-^\beta\Pi_{+m} + \frac{1}{6}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\theta_-^\beta\theta_-^\gamma\dot{\theta}_-^\delta\right] + \\ &+ \delta\theta_-^\alpha\left[(1-c_1)\,d_\alpha^+ - \frac{1}{6}\gamma^m_{\alpha\beta}\gamma_{m\gamma\delta}\theta_-^\beta\theta_-^\gamma\dot{\theta}_+^\delta\right] + \\ &+ \delta y_m\left[\widehat{\Pi}^m - \Pi^m + \theta_-\gamma^m\dot{\theta}_+\right] + \\ &+ c_1\delta d_\alpha^+\theta_-^\alpha - \delta\lambda_+^\alpha w_\alpha^- + (c_2-1)\delta\lambda_-^\alpha w_\alpha^+ + c_2\delta w_\alpha^+\lambda_-^\alpha\Big\}\,, \quad (3.16)\end{aligned}$$

$$
\begin{aligned}
\delta V = \frac{1}{2\alpha'} \int d\tau \Big\{ & \delta\theta_+^\alpha \Big[\dot\theta_+^\beta (\gamma^m_{\alpha\beta} B_m - D_\alpha A_\beta - D_\beta A_\alpha) + \Pi_+^m (D_\alpha B_m - \partial_m A_\alpha) - \\
& - d_\gamma^+ D_\alpha W^\gamma + \frac{1}{2} N_+^{mn} D_\alpha F_{mn} \Big] + \\
& + \delta y^m \Big[\dot\theta_+^\alpha (\partial_m A_\alpha - D_\alpha B_m) + \Pi_+^n (\partial_m B_n - \partial_n B_m) + \\
& + d_\gamma^+ \partial_m W^\gamma + \frac{1}{2} N_+^{nk} \partial_m F_{nk} \Big] + \\
& + \delta d_\alpha^+ W^\alpha + \frac{1}{4} (\delta\lambda_+ \gamma^{mn} w^+) F_{mn} + \frac{1}{4} (\lambda_+ \gamma^{mn} \delta w^+) F_{mn} \Big\} . \qquad (3.17)
\end{aligned}
$$

Cancelling the terms with δd_α^+ in $\delta(S_0 + S_b + V)$, we obtain the boundary condition

$$
\theta_-^\alpha = -\frac{1}{c_1} W^\alpha(x, \theta_+) , \qquad (3.18)
$$

which implies that

$$
\delta\theta_-^\alpha = -\frac{1}{c_1} \left(\delta\theta_+^\beta D_\beta W^\alpha + \delta y^m \partial_m W^\alpha \right) , \qquad \dot\theta_-^\alpha = -\frac{1}{c_1} \left(\dot\theta_+^\beta D_\beta W^\alpha + \Pi_+^m \partial_m W^\alpha \right) . \qquad (3.19)
$$

Plugging (3.19) back into (3.16), cancellation of the remaining terms in $\delta(S_0 + S_b + V)$ implies the boundary conditions

$$
\begin{aligned}
\Pi_m - \widehat\Pi_m &= \dot\theta_+^\alpha \left(\partial_m A_\alpha - D_\alpha B_m + \frac{1}{c_1} \gamma_{m\alpha\beta} W^\beta + \frac{1}{6c_1^3} \gamma^n_{\alpha\beta} \gamma_{n\gamma\delta} W^\beta W^\gamma \partial_m W^\delta \right) + \\
& + \Pi_+^n (\partial_m B_n - \partial_n B_m) + \frac{1}{c_1} d_\alpha^+ \partial_m W^\alpha + \frac{1}{2c_2} N_+^{nk} \partial_m F_{nk} , \\
\sqrt{2} \left(d_\alpha - \widehat d_\alpha \right) &= \dot\theta_+^\beta \Big(D_\alpha A_\beta + D_\beta A_\alpha - \gamma^m_{\alpha\beta} B_m + \frac{1}{6c_1^3} \gamma^n_{\alpha\gamma} \gamma_{n\delta\lambda} W^\gamma W^\delta D_\beta W^\lambda + \\
& + \frac{1}{6c_1^3} \gamma^n_{\beta\gamma} \gamma_{n\delta\lambda} W^\gamma W^\delta D_\alpha W^\lambda \Big) + \\
& + \Pi_+^m \left(\partial_m A_\alpha - D_\alpha B_m + \frac{1}{c_1} \gamma_{m\alpha\beta} W^\beta + \frac{1}{6c_1^3} \gamma^n_{\alpha\beta} \gamma_{n\gamma\delta} W^\beta W^\gamma \partial_m W^\delta \right) + \\
& + \frac{1}{c_1} d_\gamma^+ D_\alpha W^\gamma - \frac{1}{2c_2} N_+^{mn} D_\alpha F_{mn} , \\
\lambda_-^\alpha &= -\frac{1}{4c_2} \lambda_+^b (\gamma_{mn})_\beta{}^\alpha F^{mn} , \qquad w_\alpha^- = \frac{1}{4c_2} F^{mn} (\gamma_{mn})_\alpha{}^\beta w_\beta^+ . \qquad (3.20)
\end{aligned}
$$

Note that the above boundary conditions become singular when $c_1 = c_2 = 0$. However, for any non-zero value of c_1 and c_2, the dependence of the boundary conditions on c_1 and c_2 can be eliminated by rescaling $W^\alpha \to c_1 W^\alpha$ and $F_{mn} \to c_2 F_{mn}$. So without loss of generality, we will set $c_1 = c_2 = 1$ for the rest of this paper. In the following subsection, the boundary conditions of (3.18) and (3.20) will be used to obtain the effective equations of the motion for the background superfields from the requirement that $\lambda^\alpha d_\alpha = \widehat\lambda^\alpha \widehat d_\alpha$ on the boundary.

3.4 Abelian supersymmetric Born-Infeld equations

Using the boundary conditions of (3.18) and (3.20), the difference between the left and right-moving BRST currents on the boundary is

$$\begin{aligned}
2(\lambda^\alpha d_\alpha - \widehat{\lambda}^\alpha \widehat{d}_\alpha) &= \lambda_+^\alpha \sqrt{2}(d_\alpha - \widehat{d}_\alpha) + \lambda_-^\alpha d_\alpha^+ + \frac{1}{2}(\lambda_- \gamma_m \theta_-)(\widehat{\Pi}^m - \Pi^m) \\
&= \lambda_+^\alpha \dot{\theta}_+^\beta \Big[D_\alpha A_\beta + D_\beta A_\alpha - \gamma_{\alpha\beta}^m B_m + \frac{1}{6}\gamma_{\alpha\gamma}^m \gamma_{m\delta\lambda} W^\gamma W^\delta D_\beta W^\lambda + \\
&\qquad + \frac{1}{6}\gamma_{\beta\gamma}^m \gamma_{m\delta\lambda} W^\gamma W^\delta D_\alpha W^\lambda + \frac{1}{8}(\gamma F)_\alpha{}^\kappa \gamma_{\kappa\lambda}^m W^\lambda \times \\
&\qquad \times (\partial_m A_\beta - D_\beta B_m + \gamma_{m\beta\gamma} W^\gamma + \frac{1}{6}\gamma_{\beta\sigma}^n \gamma_{n\gamma\delta} W^\sigma W^\gamma \partial_m W^\delta) \Big] \\
&\quad + \lambda_+^\alpha \Pi_+^m \Big[\partial_m A_\alpha - D_\alpha B_m + \gamma_{m\alpha\beta} W^\beta + \frac{1}{6}\gamma_{\alpha\beta}^n \gamma_{n\gamma\delta} W^\beta W^\gamma \partial_m W^\delta \\
&\qquad - \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\gamma}^n W^\gamma (\partial_n B_m - \partial_m B_n) \Big] \\
&\qquad + \lambda_+^\alpha d_\gamma^+ \Big[D_\alpha W^\gamma - \frac{1}{4}(\gamma F)_\alpha{}^\gamma + \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\lambda}^n W^\lambda \partial_n W^\gamma \Big] - \\
&\qquad - \frac{1}{2}\lambda_+^\alpha N_+^{mn} \Big[D_\alpha F_{mn} + \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\lambda}^k W^\lambda \partial_k F_{mn} \Big] , \qquad (3.21)
\end{aligned}$$

where $(\gamma F)_\beta{}^\alpha = F_{mn}(\gamma^{mn})_\beta{}^\alpha$.

Requiring this to be zero implies the equations:

$$\begin{aligned}
D_\alpha A_\beta + D_\beta A_\alpha - \gamma_{\alpha\beta}^m B_m + \frac{1}{6}\gamma_{\alpha\gamma}^m \gamma_{\delta\lambda} W^\gamma W^\delta D_\beta W^\lambda + \\
+ \frac{1}{6}\gamma_{\beta\gamma}^m \gamma_{m\delta\lambda} W^\gamma W^\delta D_\alpha W^\lambda + \frac{1}{64}(\gamma F)_\alpha{}^\gamma (\gamma F)_\beta{}^\delta \gamma_{\gamma\lambda}^n \gamma_{\delta\sigma}^m W^\lambda W^\sigma (\partial_m B_n - \partial_n B_m) &= 0 , \quad (3.22) \\
\partial_m A_\alpha - D_\alpha B_m + \gamma_{m\alpha\beta} W^\beta + \frac{1}{6}\gamma_{\alpha\beta}^n \gamma_{n\gamma\delta} W^\beta W^\gamma \partial_m W^\delta - \\
- \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\lambda}^n W^\lambda (\partial_n B_m - \partial_m B_n) &= 0 , \quad (3.23) \\
D_\alpha W^\gamma - \frac{1}{4}(\gamma F)_\alpha{}^\gamma + \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\lambda}^n W^\lambda \partial_n W^\gamma &= 0 , \quad (3.24) \\
\lambda_+^\alpha \lambda_+^\beta (\gamma^{mn})_\beta{}^\gamma \Big[D_\alpha F_{mn} + \frac{1}{8}(\gamma F)_\alpha{}^\beta \gamma_{\beta\lambda}^k W^\lambda \partial_k F_{mn} \Big] &= 0 . \quad (3.25)
\end{aligned}$$

As in the super-Yang-Mills equations of (2.9), the contraction of (3.22) with $\gamma_{mnpqr}^{\alpha\beta}$ implies the equations of motion for A_α, the contraction of (3.22) with $\gamma_m^{\alpha\beta}$ defines B_m, the contraction of (3.23) with $\gamma^{m\alpha\gamma}$ defines W^γ, the contraction of (3.24) with $(\gamma_{rs})_\beta{}^\alpha$ defines F_{rs}, and the remaining contractions of (3.23) and (3.24) are implied by these equations through Bianchi identities. Note that because of the non-linear terms in (3.22)–(3.24), W^γ and F_{mn} are now complicated functions of the spinor and vector field strengths constructed from the gauge fields A_α and B_m.

Finally, equation (3.25) vanishes as a consequence of (3.24) and the pure spinor property

$$\lambda_+ \gamma^m \lambda_+ + \frac{1}{16}(\gamma F)_\gamma{}^\alpha (\gamma F)_\delta{}^\beta \gamma_{\alpha\beta}^m \lambda_+^\gamma \lambda_+^\delta = \lambda_+ \gamma^m \lambda_+ + \lambda_- \gamma^m \lambda_- = \lambda \gamma^m \lambda + \widehat{\lambda} \gamma^m \widehat{\lambda} = 0 . \quad (3.26)$$

To show that (3.25) vanishes, it is useful to write (3.24) and (3.25) as $\widehat{D}_\alpha W^\gamma = \frac{1}{4}(\gamma F)_\alpha{}^\gamma$ and $\lambda_+^\alpha \lambda_+^\beta \widehat{D}_\alpha \widehat{D}_\beta W^\gamma = 0$ where

$$\widehat{D}_\alpha = D_\alpha + \frac{1}{2} D_\alpha W^\gamma \Big(\delta_\beta^\gamma - \frac{1}{2}\gamma^n_{\beta\lambda} W^\lambda \partial_n W^\gamma \Big)^{-1} (\gamma^r W)_\beta \partial_r \,. \tag{3.27}$$

One can check that

$$\Big\{ \widehat{D}_\alpha, \widehat{D}_\beta \Big\} = \Big(\gamma^m_{\alpha\beta} + \frac{1}{16}(\gamma F)_\alpha{}^\gamma (\gamma F)_\beta{}^\delta \gamma^m_{\gamma\delta} \Big) \widehat{\partial}_m \tag{3.28}$$

where

$$\widehat{\partial}_m = \partial_m + \frac{1}{2}\partial_m W^\gamma \Big(\delta_\beta^\gamma - \frac{1}{2}\gamma^n_{\beta\lambda} W^\lambda \partial_n W^\gamma \Big)^{-1} (\gamma^r W)_\beta \partial_r \,, \tag{3.29}$$

so (3.26) implies that $\lambda_+^\alpha \lambda_+^\beta \widehat{D}_\alpha \widehat{D}_\beta W^\gamma = 0$.

To prove that equations (3.22)–(3.24) are the abelian supersymmetric Born-Infeld equations, it will now be shown that they are invariant under $N = 2$ $D = 10$ supersymmetry where the second supersymmetry acts non-linearly on the superfields. Except for factors of i coming from different conventions for the supersymmetry algebra, equations (3.22) - (3.24) are easily shown to coincide with the superspace Born-Infeld equations (33)-(35) of reference [4] which were independently derived using the superembedding method [9].

3.5 Non-linearly realized supersymmetry

In addition to the supersymmetry parameterized by $\epsilon_+ = \frac{1}{\sqrt{2}}(\epsilon^\alpha + \widehat{\epsilon}^\alpha)$ in (3.10), the closed superstring action of (3.2) has a second supersymmetry parameterized by $\epsilon_- = \frac{1}{\sqrt{2}}(\epsilon^\alpha - \widehat{\epsilon}^\alpha)$ where

$$\delta_{\epsilon_-} \theta_+^\alpha = 0 \,, \quad \delta_{\epsilon_-} \theta_-^\alpha = \epsilon_-^\alpha \,, \quad \delta_{\epsilon_-} x^m = \frac{1}{2}\theta_- \gamma^m \epsilon_- \,, \tag{3.30}$$

and the transformation $\delta_{\epsilon_-} p_\alpha^\pm$ can be determined from (3.3). Under this second supersymmetry, $S_0 + S_b$ is not invariant and transforms as

$$\begin{aligned} \delta_{\epsilon_-}(S_0 + S_b) = \frac{1}{2\alpha'} \int d\tau \, \Big(& \frac{1}{2}(\epsilon_- \gamma^m \theta_-) \Big(\Pi_m - \widehat{\Pi}_m \Big) + d_\alpha^+ \epsilon_-^\alpha - (\epsilon_- \gamma_m \theta_+) \, \Pi_+^m + \\ & + \frac{1}{3}(\epsilon_- \gamma^m \theta_+)(\theta_+ \gamma_m \dot{\theta}_+) + \frac{1}{6}(\epsilon_- \gamma^m \theta_-) \Big(\dot{\theta}_+ \gamma_m \theta_- \Big) \Big). \end{aligned} \tag{3.31}$$

Note that even for the free boundary condition $\theta_-^\alpha = 0$, this variation does not vanish. However, by suitably transforming the background superfields $[A_\alpha, B_m, W^\alpha, F_{mn}]$ in a non-linear manner, the variation of (3.31) can be cancelled by the variation of the vertex operator V. Since the BRST currents $\lambda^\alpha d_\alpha$ and $\widehat{\lambda}^\alpha \widehat{d}_\alpha$ are invariant under supersymmetry transformations parameterized by both ϵ_+^α and ϵ_-^α, the equations of (3.22)–(3.24) coming from classical BRST invariance of the action are guaranteed to be invariant under this non-linearly realized supersymmetry transformation of the background superfields.

To find the explicit form of the non-linear supersymmetry transformation, note that

$$\begin{aligned}\delta_{\epsilon_-} V = \frac{1}{2\alpha'} \int d\tau\, \Big\{ & \dot\theta^\alpha_+ \Big(\delta_{\epsilon_-} A_\alpha + \frac{1}{2}(\epsilon_-\gamma^m W)\,\partial_m A_\alpha\Big) + \Pi^m_+ \Big(\delta_{\epsilon_-} B_m + \frac{1}{2}(\epsilon_-\gamma^n W)\,\partial_n B_m\Big) \\ & + d^+_\alpha \Big(\delta_{\epsilon_-} W^\alpha + \frac{1}{2}(\epsilon_-\gamma^m W)\,\partial_m W^\alpha\Big) + \\ & + \frac{1}{2} N^{mn}_+ \Big(\delta_{\epsilon_-} F_{mn} + \frac{1}{2}(\epsilon_-\gamma^k W)\,\partial_k F_{mn}\Big) + \\ & + \frac{1}{2}(\epsilon_-\gamma^m\theta_-)(\dot\theta^\alpha_+ D_\alpha B_m + \Pi^n_+ \partial_n B_m) + \frac{1}{2}(\epsilon_-\gamma^m W)(\Pi_m - \widehat\Pi_m)\Big\} \end{aligned} \tag{3.32}$$

where the terms $\frac{1}{2}(\epsilon_-\gamma^m W)\partial_m$ in (3.32) come from the transformation of x^m in (3.30), the term $\frac{1}{2}(\epsilon_-\gamma^m\theta_-)(\dot\theta^\alpha_+ D_\alpha B_m + \Pi^n_+\partial_n B_m)$ comes from integrating by parts the term $(\delta_{\epsilon_-}\Pi^m_+)B_m$, and the term $\frac{1}{2}(\epsilon_-\gamma^m W)(\Pi_m - \widehat\Pi_m)$ comes from $(\delta_{\epsilon_-} d^+_\alpha)W^\alpha$. Requiring that $\delta_{\epsilon_-}(S_0 + S_b + V) = 0$, one finds

$$\begin{aligned} \delta_{\epsilon_-} A_\alpha &= \frac{1}{3}\gamma_{m\alpha\beta}\theta^\beta_+\,(\epsilon_-\gamma^m\theta_+) - \frac{1}{2}(\epsilon_-\gamma^m W)\left(\partial_m A_\alpha - D_\alpha B_m + \frac{1}{3}\gamma_{m\alpha\beta}W^\beta\right), \\ \delta_{\epsilon_-} B_m &= (\epsilon_-\gamma_m\theta_+) - \frac{1}{2}(\epsilon_-\gamma^n W)(\partial_n B_m - \partial_m B_n)\,, \\ \delta_{\epsilon_-} W^\gamma &= -\epsilon^\gamma_- - \frac{1}{2}(\epsilon_-\gamma^m W)\,\partial_m W^\gamma\,, \qquad \delta_{\epsilon_-} F_{mn} = -\frac{1}{2}(\epsilon_-\gamma^k W)\,\partial_k F_{mn}\,. \end{aligned} \tag{3.33}$$

It is straightforward to check that the transformations of (3.33) leave the supersymmetric Born-Infeld equations of (3.22)–(3.24) invariant and that they combine with the manifest $N = 1$ $D = 10$ supersymmetry transformations parameterized by ϵ_+ to form the $N = 2$ $D = 10$ algebra

$$\left[\delta_{\epsilon^1_-}, \delta_{\epsilon^2_-}\right] = (\epsilon^2_-\gamma^m\epsilon^1_-)\,\partial_m\,, \quad \left[\delta_{\epsilon^1_+}, \delta_{\epsilon^2_+}\right] = (\epsilon^2_+\gamma^m\epsilon^1_+)\,\partial_m\,, \quad \left[\delta_{\epsilon^1_-}, \delta_{\epsilon^2_+}\right] = 0, \tag{3.34}$$

up to a gauge transformation of A_α and B_m.

4. Open superstring in a non-abelian background

In this section we will generalize the results of the previous section for the case of a U(N) non-abelian background. The superfields belonging to the U(1) abelian and SU(N) non-abelian subgroups will be denoted as $[A_\alpha, B_m, W^\alpha, F_{mn}]$ and $[\widehat{A}_{\alpha I}{}^J, \widehat{B}_{mI}{}^J, \widehat{W}^{\alpha J}_I, \widehat{F}_{mnI}{}^J]$ where $I, J = 1$ to N and the hatted superfields are traceless in these indices.

As in the superparticle action of (2.4), interaction with the non-abelian background can be described at the classical level by introducing complex worldline fermionic fields η_I and $\bar\eta^I$ on the boundary and defining the vertex as

$$\begin{aligned} V = \frac{1}{2\alpha'}\int d\tau\,\Big\{ & \dot\theta^\alpha_+ A_\alpha(x,\theta_+) + \Pi^m_+ B_m(x,\theta_+) + d^+_\alpha W^\alpha(x,\theta_+) + \frac{1}{2}N^{mn}_+ F_{mn}(x,\theta_+) + \\ & + \bar\eta^I\dot\eta_I + \bar\eta^I\Big(\dot\theta^\alpha_+\widehat{A}_\alpha(x,\theta_+) + \Pi^m_+\widehat{B}_m(x,\theta_+) + \\ & \qquad + d^+_\alpha\widehat{W}^\alpha(x,\theta_+) + \frac{1}{2}N^{mn}_+\widehat{F}_{mn}(x,\theta_+)\Big)_I{}^J\eta_J\Big\}\,. \end{aligned} \tag{4.1}$$

Since $\bar{\eta}^I$ and η_I carry dimension -1 and $[\theta^\alpha, x^m, p_\alpha, N^{mn}]$ carry dimension $[-\frac{1}{2}, -1, -\frac{3}{2}, -2]$, the abelian superfields $[A_\alpha, B_m, W^\alpha, F_{mn}]$ carry dimension $[-\frac{3}{2}, -1, -\frac{1}{2}, 0]$ and the non-abelian superfields $[\widehat{A}_\alpha, \widehat{B}_m, \widehat{W}^\alpha, \widehat{F}_{mn}]$ carry dimension $[\frac{1}{2}, 1, \frac{3}{2}, 2]$. As explained in the introduction, this different definition of dimension for abelian and non-abelian background fields allows a consistent α' expansion. As will be seen in this section, the lowest-order contribution to the abelian equations of motion will be unaffected by the non-abelian fields but the lowest-order contribution to the non-abelian equations of motion will include corrections to the super-Yang-Mills equations coming from couplings to the abelian field strength.

4.1 Boundary conditions in a non-abelian background

Using the vertex operator of (4.1), the surface term variation of the full action is

$$
\begin{aligned}
\delta(S_0 + S_b + V) = \frac{1}{2\alpha'} \int d\tau \Bigg\{ & \delta\bar{\eta}\left[\dot{\eta} + \left(\dot{\theta}_+^\alpha \widehat{A}_\alpha + \Pi_+^m \widehat{B}_m + d_\alpha^+ \widehat{W}^\alpha + \frac{1}{2} N_+^{mn} \widehat{F}_{mn}\right)\eta\right] + \\
& + \left[-\dot{\bar{\eta}} + \bar{\eta}\left(\dot{\theta}_+^\alpha \widehat{A}_\alpha + \Pi_+^m \widehat{B}_m + d_\alpha^+ \widehat{W}^\alpha + \frac{1}{2} N_+^{mn} \widehat{F}_{mn}\right)\right]\delta\eta + \\
& + \delta\theta_+^\alpha \Big[\sqrt{2}\left(d_\alpha - \widehat{d}_\alpha\right) + \gamma_{\alpha\beta}^m \theta_-^\beta \Pi_{+m} + \frac{1}{6}\gamma_{\alpha\beta}^m \gamma_{m\gamma\delta}\theta_-^\beta \theta_-^\gamma \dot{\theta}_-^\delta + \\
& \qquad + \dot{\bar{\eta}}\widehat{A}_\alpha \eta + \bar{\eta}\widehat{A}_\alpha \dot{\eta} + \bar{\eta}\dot{\theta}_+^\beta\left(D_\alpha \widehat{A}_\beta + D_\beta \widehat{A}_\alpha - \gamma_{\alpha\beta}^m \widehat{B}_m\right)\eta + \\
& \qquad + \bar{\eta}\Pi_+^m\left(\partial_m \widehat{A}_\alpha - D_\alpha \widehat{B}_m\right) + \bar{\eta} d_\beta^+ D_\alpha \widehat{W}^\beta \eta - \\
& \qquad - \frac{1}{2}\bar{\eta} N_+^{mn} D_\alpha \widehat{F}_{mn}\eta + \dot{\theta}_+^\beta\left(\gamma_{\alpha\beta}^m B_m - D_\alpha A_\beta - D_\beta A_\alpha\right) + \\
& \qquad + \Pi_+^m (D_\alpha B_m - \partial_m A_\alpha) - d_\beta^+ D_\alpha W^\beta + \frac{1}{2} N_+^{mn} D_\alpha F_{mn}\Big] + \\
& + \delta\theta_-^\alpha\left[-\frac{1}{6}\gamma_{\alpha\beta}^m \gamma_{m\gamma\delta}\theta_-^\beta \theta_-^\gamma \dot{\theta}_+^\delta\right] + \\
& + \delta y^m \Big[\widehat{\Pi}_m - \Pi_m + \theta_- \gamma_m \dot{\theta}_+ - \dot{\bar{\eta}}\widehat{B}_m \eta - \bar{\eta}\widehat{B}_m \dot{\eta} + \\
& \qquad + \bar{\eta}\dot{\theta}_+^\alpha\left(\partial_m \widehat{A}_\alpha - D_\alpha \widehat{B}_m\right)\eta + \bar{\eta}\Pi_+^n\left(\partial_m \widehat{B}_n - \partial_n \widehat{B}_m\right)\eta + \\
& \qquad + \bar{\eta} d_\beta^+ \partial_m \widehat{W}^\beta \eta + \frac{1}{2}\bar{\eta} N_+^{nk}\partial_m \widehat{F}_{nk}\eta \\
& \qquad + \dot{\theta}_+^\alpha (\partial_m A_\alpha - D_\alpha B_m) + \Pi_+^n (\partial_m B_n - \partial_n B_m) \\
& \qquad + d_\beta^+ \partial_m W^\beta + \frac{1}{2} N_+^{nk}\partial_m F_{nk}\Big] + \\
& + \delta d_\alpha^+ \left[\theta_-^\alpha - \bar{\eta}\widehat{W}^\alpha \eta + W^\alpha\right] + \\
& + \delta w_\alpha^+ \left[\lambda_-^\alpha + \frac{1}{4}\lambda_+^\beta \gamma^{mn}{}_\beta{}^\alpha \left(\bar{\eta}\widehat{F}_{mn}\eta + F_{mn}\right)\right] + \\
& + \delta\lambda_+^\alpha \left[-w_\alpha^- + \frac{1}{4}\gamma^{mn}{}_\alpha{}^\beta w_\beta^+ \left(\bar{\eta}\widehat{F}_{mn}\eta + F_{mn}\right)\right]\Bigg\}, \qquad (4.2)
\end{aligned}
$$

where $c_1 = c_2 = 1$ in S_b of (3.12).

As in the previous section, the variations of δd^+_α, δw^+_α and $\delta\lambda^\alpha_+$ can be cancelled by choosing the boundary conditions

$$
\begin{aligned}
\theta^\alpha_- &= \bar\eta\widehat{W}^\alpha\eta - W^\alpha, \\
\lambda^\alpha_- &= -\frac{1}{4}\lambda^\beta_+\gamma^{mn}{}_\beta{}^\alpha\left(\bar\eta\widehat{F}_{mn}\eta + F_{mn}\right), \qquad w^-_\alpha = \frac{1}{4}\gamma^{mn}{}_\alpha{}^\beta w^+_\beta\left(\bar\eta\widehat{F}_{mn}\eta + F_{mn}\right).
\end{aligned}
\tag{4.3}
$$

Plugging θ^α_- back into (4.2) produces terms up to sixth order in η. However, as will be seen later, boundary conditions independent of η will contribute to the lowest-order abelian equations of motion while boundary conditions quadratic in η will contribute to the lowest-order non-abelian equations of motion. Since boundary conditions involving more than two η's do not contribute to these equations at the lowest order in α', they can be ignored in the following discussion. However, as will be discussed in the concluding section, these higher-order terms in η will be relevant for computing higher-derivative corrections to the Born-Infeld equations.

Cancellation of the terms proportional to $\delta\bar\eta^I$, $\delta\eta_I$, δy^m and $\delta\theta^\alpha_+$ in (4.2) implies the following boundary conditions up to quadratic order in η:

$$
\begin{aligned}
\dot\eta &= -\left[\dot\theta^\alpha_+\left(\widehat{A}_\alpha - \frac{1}{6}\gamma^m_{\alpha\delta}\gamma_{m\beta\gamma}\widehat{W}^\beta W^\gamma W^\delta\right) + \Pi^m_+\widehat{B}_m + d^+_\alpha\widehat{W}^\alpha + \frac{1}{2}N^{mn}_+\widehat{F}_{mn}\right]\eta \\
\dot{\bar\eta} &= \bar\eta\left[\dot\theta^\alpha_+\left(\widehat{A}_\alpha - \frac{1}{6}\gamma^m_{\alpha\delta}\gamma_{m\beta\gamma}\widehat{W}^\beta W^\gamma W^\delta\right) + \Pi^m_+\widehat{B}_m + d^+_\alpha\widehat{W}^\alpha + \frac{1}{2}N^{mn}_+\widehat{F}_{mn}\right] \\
\Pi_m - \widehat\Pi_m &= \bar\eta\dot\theta^\alpha_+\Big(\partial_m\widehat{A}_\alpha - D_\alpha\widehat{B}_m + \left[\widehat{B}_m, \widehat{A}_\alpha\right] + \gamma_{m\alpha\beta}\widehat{W}^\beta + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}\widehat{W}^\beta W^\gamma\partial_m W^\delta + \\
&\quad + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta\widehat{W}^\gamma\partial_m W^\delta + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta W^\gamma\nabla_m\widehat{W}^\delta\Big)\eta + \\
&\quad + \bar\eta\Pi^n_+\left(\partial_m\widehat{B}_n - \partial_n\widehat{B}_m + \left[\widehat{B}_m, \widehat{B}_n\right]\right)\eta + \bar\eta d^+_\alpha\nabla_m\widehat{W}^\alpha\eta + \frac{1}{2}\bar\eta N^{nk}_+\nabla_m\widehat{F}_{nk}\eta + \\
&\quad + \dot\theta^\alpha_+\left(\partial_m A_\alpha - D_\alpha B_m + \gamma_{m\alpha\beta}W^\beta + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta W^\gamma\partial_m W^\delta\right) + \\
&\quad + \Pi^n_+(\partial_m B_n - \partial_n B_m) + d^+_\alpha\partial_m W^\alpha + \frac{1}{2}N^{nk}_+\partial_m F_{nk}\,, \\
\sqrt{2}\left(d_\alpha - \widehat{d}_\alpha\right) &= -\bar\eta\dot\theta^\beta_+\Big(D_\alpha\widehat{A}_\beta + D_\beta\widehat{A}_\alpha + \left\{\widehat{A}_\alpha, \widehat{A}_\beta\right\} - \gamma^m_{\alpha\beta}\widehat{B}_m + \frac{1}{3}\gamma^m_{\epsilon\gamma}\gamma_{m\delta(\alpha}\nabla_{\beta)}\widehat{W}^\gamma W^\delta W^\epsilon + \\
&\quad + \frac{1}{3}\gamma^m_{\epsilon\gamma}\gamma_{m\delta(\alpha}D_{\beta)}W^\gamma\widehat{W}^\delta W^\epsilon + \frac{1}{3}\gamma^m_{\epsilon\gamma}\gamma_{m\delta(\alpha}D_{\beta)}W^\gamma W^\delta\widehat{W}^\epsilon - \\
&\quad - \frac{1}{36}\gamma^m_{\alpha\gamma}\gamma_{m\delta\epsilon}\,\gamma^n_{\beta\rho}\gamma_{n\sigma\pi}W^\gamma W^\delta W^\sigma W^\rho\left\{\widehat{W}^\pi, \widehat{W}^\epsilon\right\}\Big)\eta - \\
&\quad - \bar\eta\Pi^m_+\Big(\partial_m\widehat{A}_\alpha - D_\alpha\widehat{B}_m + \left[\widehat{B}_m, \widehat{A}_\alpha\right]\gamma_{m\alpha\beta}\widehat{W}^\beta + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}\widehat{W}^\beta W^\gamma\partial_m W^\delta + \\
&\quad + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta\widehat{W}^\gamma\partial_m W^\delta + \frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta W^\gamma\nabla_m\widehat{W}^\delta\Big)\eta - \\
&\quad - \bar\eta d^+_\beta\left(\nabla_\alpha\widehat{W}^\beta + \frac{1}{6}\gamma^m_{\alpha\gamma}\gamma_{m\delta\epsilon}W^\gamma W^\delta\left\{\widehat{W}^\beta, \widehat{W}^\epsilon\right\}\right)\eta - \\
&\quad + \frac{1}{2}\bar\eta N^{mn}_+\left(\nabla_\alpha\widehat{F}_{mn} - \frac{1}{6}\gamma^k_{\alpha\beta}\gamma_{k\gamma\delta}W^\beta W^\gamma\left[\widehat{F}_{mn}, \widehat{W}^\delta\right]\right)\eta +
\end{aligned}
$$

$$
\begin{aligned}
&+\dot\theta_+^\beta\left(D_\alpha A_\beta+D_\beta A_\alpha-\gamma^m_{\alpha\beta}B_m+\frac{1}{3}\gamma^m_{\epsilon\gamma}\gamma_{m\delta(\alpha}D_{\beta)}W^\gamma W^\delta W^\epsilon\right)\\
&+\Pi_+^m\left(\partial_m A_\alpha-D_\alpha B_m+\gamma_{m\alpha\beta}W^\beta+\frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta W^\gamma\partial_m W^\delta\right)+\\
&+d^+_\beta D_\alpha W^\beta-\frac{1}{2}N_+^{mn}D_\alpha F_{mn}\,,
\end{aligned}
\tag{4.4}
$$

where $\nabla_\alpha=D_\alpha+\widehat{A}_\alpha$ and $\nabla_m=\partial_m+\widehat{B}_m$.

4.2 Non-abelian equations of motion

As in the previous section, the equations of motion for the background superfields are obtained by requiring that $\lambda^\alpha d_\alpha=\widehat{\lambda}^\alpha\widehat{d}_\alpha$ on the boundary using the boundary conditions of (4.3) and (4.4). Writing

$$
2\left(\lambda^\alpha d_\alpha-\widehat{\lambda}^\alpha\widehat{d}_\alpha\right)=\lambda_+^\alpha\sqrt{2}\left(d_\alpha-\widehat{d}_\alpha\right)+\lambda_-^\alpha d_\alpha^+ +\frac{1}{2}(\lambda_-\gamma_m\theta_-)\left(\widehat{\Pi}^m-\Pi^m\right),[1pt]
\tag{4.5}
$$

one can easily check that the vanishing of η-independent terms in (4.5) implies the same abelian Born-Infeld equations (3.22)–(3.24) as in the previous section. And requiring the vanishing of terms quadratic in η in (4.5) implies the non-abelian equations

$$
\begin{aligned}
&D_\alpha\widehat{A}_\beta+D_\beta\widehat{A}_\alpha+\left\{\widehat{A}_\alpha,\widehat{A}_\beta\right\}-\gamma^m_{\alpha\beta}\widehat{B}_m+\frac{1}{3}\gamma^m_{\epsilon\delta}\gamma_{m\gamma(\alpha}\nabla_{\beta)}\widehat{W}^\delta W^\gamma W^\epsilon+\\
&+\frac{1}{3}\gamma^m_{\epsilon\delta}\gamma_{m\gamma(\alpha}D_{\beta)}W^\delta\widehat{W}^\gamma W^\epsilon+\frac{1}{3}\gamma^m_{\epsilon\delta}\gamma_{m\gamma(\alpha}D_{\beta)}W^\delta W^\gamma\widehat{W}^\epsilon-\\
&-\frac{1}{36}\gamma^m_{\alpha\gamma}\gamma_{m\delta\epsilon}\;\gamma^n_{\beta\rho}\gamma_{n\sigma\pi}W^\gamma W^\delta W^\sigma W^\rho\{\widehat{W}^\pi,\widehat{W}^\epsilon\}+\\
&+\frac{1}{64}\Big(\left(\gamma\widehat{F}\right)_\alpha{}^\gamma(\gamma F)_\beta{}^\delta W^\rho W^\sigma+(\gamma F)_\alpha{}^\gamma(\gamma\widehat{F})_\beta{}^\delta W^\rho W^\sigma+\\
&+(\gamma F)_\alpha{}^\gamma(\gamma F)_\beta{}^\delta\widehat{W}^\rho W^\sigma+\;+(\gamma F)_\alpha{}^\gamma(\gamma F)_\beta{}^\delta W^\rho\widehat{W}^\sigma\Big)\gamma^n_{\gamma\rho}\gamma^m_{\delta\sigma}(\partial_m B_n-\partial_n B_m)+\\
&+\frac{1}{64}(\gamma F)_\alpha{}^\gamma(\gamma F)_\beta{}^\delta W^\rho W^\sigma\gamma^n_{\gamma\rho}\gamma^m_{\delta\sigma}(\partial_m\widehat{B}_n-\partial_n\widehat{B}_m+[\widehat{B}_m,\widehat{B}_n])=0,
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
&\partial_m\widehat{A}_\alpha-D_\alpha\widehat{B}_m+\left[\widehat{B}_m,\widehat{A}_\alpha\right]+\gamma_{m\alpha\beta}\widehat{W}^\beta+\\
&+\frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}\widehat{W}^\beta W^\gamma\partial_m W^\delta+\frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta\widehat{W}^\gamma\partial_m W^\delta+\\
&+\frac{1}{6}\gamma^n_{\alpha\beta}\gamma_{n\gamma\delta}W^\beta W^\gamma\nabla_m\widehat{W}^\delta-\frac{1}{8}\left(\left(\gamma\widehat{F}\right)_\alpha{}^\beta W^\gamma+(\gamma F)_\alpha{}^\beta\widehat{W}^\gamma\right)\gamma^n_{\beta\gamma}(\partial_n B_m-\partial_m B_n)-\\
&-\frac{1}{8}(\gamma F)_\alpha{}^\beta W^\gamma\gamma^n_{\beta\gamma}\left(\partial_n\widehat{B}_m-\partial_m\widehat{B}_n+\left[\widehat{B}_n,\widehat{B}_m\right]\right)=0,
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
&\nabla_\alpha\widehat{W}^\beta-\frac{1}{4}\left(\gamma\widehat{F}\right)_\alpha{}^\beta+\frac{1}{6}\gamma^m_{\alpha\gamma}\gamma_{m\delta\epsilon}W^\gamma W^\epsilon\left\{\widehat{W}^\beta,\widehat{W}^\delta\right\}+\\
&+\frac{1}{8}\left(\left(\gamma\widehat{F}\right)_\alpha{}^\gamma W^\delta+(\gamma F)_\alpha{}^\gamma\widehat{W}^\delta\right)\gamma^m_{\gamma\delta}\partial_m W^\beta+\\
&+\frac{1}{8}(\gamma F)_\alpha{}^\gamma W^\delta\gamma^m_{\gamma\delta}\nabla_m\widehat{W}^\beta=0,
\end{aligned}
\tag{4.8}
$$

$$
\begin{aligned}
&\lambda_+^\alpha \lambda_+^\beta (\gamma^{mn})_\beta{}^\gamma \left(\nabla_\alpha \widehat{F}_{mn} - \frac{1}{6}\gamma^m_{\alpha\delta}\gamma_{m\epsilon\sigma} W^\delta W^\epsilon \left[\widehat{F}_{mn}, \widehat{W}^\sigma\right] + \right. \\
&\left. + \frac{1}{8}\left(\left(\gamma \widehat{F}\right)_\alpha{}^\delta W^\epsilon + (\gamma F)_\alpha{}^\delta \widehat{W}^\epsilon \right) \gamma^k_{\delta\epsilon} \partial_k F_{mn} + \frac{1}{8}(\gamma F)_\alpha{}^\delta W^\sigma \gamma^k_{\delta\sigma} \nabla_k \widehat{F}_{mn} \right) = 0. \quad (4.9)
\end{aligned}
$$

As in the super-Yang-Mills and abelian Born-Infeld equations, the $\gamma^{\alpha\beta}_{mnpqr}$ contraction of (4.6) implies the equation of motion for $\widehat{A}_\alpha$, the $\gamma^{\alpha\beta}_m$ contraction of (4.6) defines $\widehat{B}_m$, the $\gamma^{m\alpha\gamma}$ contraction of (4.7) defines $\widehat{W}^\gamma$, the $(\gamma^{mn})_\beta{}^\alpha$ contraction of (4.8) defines $\widehat{F}^{mn}$, and all other contractions of (4.7)–(4.9) are implied to vanish through Bianchi identities.

4.3 Non-linearly realized supersymmetry

Just as the abelian equations of (3.22)–(3.25) are invariant under the non-linearly realized supersymmetry transformation of (3.33), the non-abelian equations of (4.6)–(4.9) are also invariant under a non-linearly realized supersymmetry transformation. This second supersymmetry can be found in the same way as in the abelian case, i.e. by requiring the total classical action $S_0 + S_b + V$ be invariant under (3.30).

Using $\delta_{\epsilon_-}(S_0 + S_b)$ from (3.31) and

$$
\delta_{\epsilon_-} V = \frac{1}{2\alpha'}\int d\tau \left\{ \dot{\theta}^\alpha_+ \left(\delta_{\epsilon_-} A_\alpha + \frac{1}{2}\left(\epsilon_- \gamma^m \left(W - \bar{\eta}\widehat{W}\eta \right)\right) \partial_m A_\alpha \right) + \dots \right\} \qquad (4.10)
$$

where the terms $\frac{1}{2}(\epsilon_-\gamma^m(W - \bar{\eta}\widehat{W}\eta))\partial_m$ in $\delta_{\epsilon_-} V$ come from $\delta_{\epsilon_-} x^m$, one finds from $\delta_{\epsilon_-}(S_0 + S_b + V) = 0$ that the abelian superfields transform as in (3.33) and the non-abelian superfields transform as

$$
\begin{aligned}
\delta_{\epsilon_-} \widehat{A}_\alpha &= -\frac{1}{2}(\epsilon_-\gamma^m W)\left(\partial_m \widehat{A}_\alpha - D_\alpha \widehat{B}_m + \left[\widehat{B}_m, \widehat{A}_\alpha\right] + \frac{1}{3}\gamma_{m\alpha\beta}\widehat{W}^\beta \right) \\
&\quad -\frac{1}{2}\left(\epsilon_-\gamma^m \widehat{W}\right)\left(\partial_m A_\alpha - D_\alpha B_m + \frac{1}{3}\gamma_{m\alpha\beta} W^\beta \right) + \nabla_\alpha \widehat{\Lambda}, \\
\delta_{\epsilon_-} \widehat{B}_m &= -\frac{1}{2}(\epsilon_-\gamma^k W)\left(\partial_k \widehat{B}_m - \partial_m \widehat{B}_k + \left[\widehat{B}_k, \widehat{B}_m\right]\right) - \\
&\quad -\frac{1}{2}\left(\epsilon_-\gamma^n \widehat{W}\right)(\partial_k B_m - \partial_m B_k) + \nabla_m \widehat{\Lambda}\,, \\
\delta_{\epsilon_-} \widehat{W}^\beta &= -\frac{1}{2}(\epsilon_-\gamma^m W)\nabla_m \widehat{W}^\beta - \frac{1}{2}\left(\epsilon_-\gamma^m \widehat{W}\right)\partial_m W^\beta + \left[\widehat{W}^\beta, \widehat{\Lambda}\right], \\
\delta_{\epsilon_-} \widehat{F}_{mn} &= -\frac{1}{2}(\epsilon_-\gamma^k W)\nabla_k \widehat{F}_{mn} - \frac{1}{2}\left(\epsilon_-\gamma^k \widehat{W}\right)\partial_k F_{mn} + \left[\widehat{F}_{mn}, \widehat{\Lambda}\right], \qquad (4.11)
\end{aligned}
$$

where $\widehat{\Lambda} = -\frac{1}{2}(\epsilon_-\gamma^m W)\widehat{B}_m$. As in the abelian case, one can check that the non-linearly realized supersymmetry transformation of (3.33) and (4.11) leave the non-abelian equations of (4.6)–(4.9) invariant and combine with the manifest $N = 1$ $D = 10$ supersymmetry transformations parameterized by ϵ_+ to form an $N = 2$ $D = 10$ supersymmetry algebra up to a gauge transformation.

5. Conclusions and higher-derivative corrections

It was shown in this paper that classical BRST invariance using the pure spinor formalism of the open superstring implies that the background satisfies the supersymmetric Born-Infeld equations of motion. These equations were expressed in $N = 1$ $D = 10$ superspace and the

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abelian contribution to these equations agrees with the results of [4]. The non-abelian contribution to these equations is new and includes corrections to the non-abelian super-Yang-Mills equations coming from coupling to the abelian field strength. In addition to the manifest $N=1$ $D=10$ supersymmetry, both the abelian and non-abelian Born-Infeld equations of motion are invariant under a non-linearly realized second supersymmetry which is related to the $N=2$ $D=10$ supersymmetry of the closed superstring worldsheet action.

Since these supersymmetric Born-Infeld equations are implied by classical BRST invariance and since the pure spinor formalism of the superstring is easy to quantize, it is natural to suppose that quantum BRST invariance implies higher-derivative corrections to these equations. In a purely abelian background, these corrections should be straightforward to compute by separating the worldsheet variables into classical and quantum variables, integrating over the quantum variables, and computing α' quantum corrections to the BRST currents $\lambda^\alpha d_\alpha$ and $\widehat{\lambda}^\alpha \widehat{d}_\alpha$. Setting $\lambda^\alpha d_\alpha = \widehat{\lambda}^\alpha \widehat{d}_\alpha$ on the boundary at the quantum level should imply higher-derivative corrections to the abelian supersymmetric Born-Infeld equations of (3.22)–(3.24).

In a non-abelian background, there is a subtlety in computing quantum corrections to the BRST currents since the classical non-abelian vertex operator V of (4.1) only involves terms with up to two η's. But after integrating over the quantum worldsheet variables, the effective vertex operator will in general contain quantum corrections involving terms quartic and higher in η. So consistency of the quantum theory implies that the vertex operator V should contain all possible couplings with even powers of η, i.e. for N real worldline fermions,

$$V = \frac{1}{2\alpha'} \int d\tau \left\{ \left(\dot{\theta}^\alpha_+ A_\alpha + \ldots \right) + \eta_I \eta_J \left(\dot{\theta}^\alpha_+ A^{IJ}_\alpha + \ldots \right) + \eta_I \eta_J \eta_K \eta_L \left(\dot{\theta}^\alpha_+ A^{IJKL}_\alpha + \ldots \right) + \ldots \right\}. \tag{5.1}$$

As discussed in [18], the elements $(1, \eta_I\eta_J, \eta_I\eta_J\eta_K\eta_L, \ldots)$ can be interpreted as even products of gamma-matrices since canonical quantization implies that $\{\eta_I, \eta_J\} = 2\delta_{IJ}$. These 2^{N-1} elements parameterize the Lie algebra $\mathrm{U}(2^{\frac{1}{2}(N-1)})$ when N is odd and the Lie algebra $\mathrm{U}(2^{\frac{1}{2}(N-2)}) \times \mathrm{U}(2^{\frac{1}{2}(N-2)})$ when N is even. So the background superfields $[A_\alpha, A^{IJ}_\alpha, A^{IJKL}_\alpha, \ldots]$ are super-Yang-Mills superfields with gauge group $\mathrm{U}(2^{\frac{1}{2}(N-1)})$ when N is odd and with gauge group $\mathrm{U}(2^{\frac{1}{2}(N-2)}) \times \mathrm{U}(2^{\frac{1}{2}(N-2)})$ when N is even. Although only an $\mathrm{SO}(N) \times \mathrm{U}(1)$ subgroup of this gauge group will be manifest in the computation, the vertex operator of (5.1) can be used to compute higher-derivative corrections to the non-abelian Born-Infeld equations. Of course, the boundary conditions involving more than two η's which were ignored in section 4 cannot be ignored in these higher-order computations.

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